Visiting LFCS over the past 25 years

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Happy 30th Birthday LFCS !

Visiting LFCS over 25 years

- It has been a great honour and privilege to frequently visit LFCS over many years and I greatly appreciate the kindness and collegiality of colleagues here.
- My first sabbatical visit to LFCS was in 1991-92.
- Memories of the early visits:
  - Constantly getting lost in Kings Buildings and ending up back at George Cleland's office.
  - Great lab barbecues for visitors and grad students.
  - Extraordinary researchers: staff, grad students, postdocs, visitors, in a very wide range of areas, from practical engineering to abstract pure maths.

Visiting LFCS: 1991-92

- More memories of the early visits:
  - The production of a coveted collection of yellow and green-covered preprints, theses, monographs. All visitors to LFCS would rush to the preprint room to grab handfuls. These preprints were a major influence world-wide on the foundations of CS.
  - Curious tradition of being disjoint from the maths department upstairs: which we tried to overcome.
- Graduate students at the time who became important scientists in their own right: including T. Altenkirch, M. Fiore, P. Gardner, N. Ghani, M. Hofmann, J. Longley, E. Moggi, D. Pym, D. Sangiorgi, A. Simpson.
- Two members of the Australian/Canadian Category Theory community became researchers here: Barry Jay, John Power.

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Logics and Categories

- LFCS became a major center for developing and applying category theory both in foundations of mathematics as well as foundations of computing.
- For many category theorists, LFCS became one of the centers of CT (along with Montreal, Sydney).
- Many contributions to CT in the history of LFCS: Rod Burstall (with Joe Goguen), Mike Fourman, Gordon Plotkin and his many students and collaborators and postdocs who used CT; later enhanced with Samson Abramsky and his students and postdocs.

Proofs-as-Processes: discussions with Milner & Abramsky

- Translating proofs in linear logic into (synchronous) Pi-calculus terms + operational semantics.
- Many discussions with Milner. Then G. Bellin (LFCS postdoc) and I studied coding LL proofs into Pi-Calculus terms.
- That year Samson visited LFCS: an important talk *Proofs-as-Processes*, and a general program outlined.
- Led to: *On the Pi-Calculus and Linear Logic* (with P.S. + G. Bellin) in TCS (1994), with an Introductory Paper, by Samson in same volume.
- Studied: Abramsky & Milner translations, information flow (I/O nets), soundness, local fullness for MLL, MALL, LL.
- Applications: Jacques Fleuriot and P. Papapanagiotou, et. al. Formal Modelling and Verification for Healthcare 2014.
- The ideas also studied later by F. von Breugel, A.Murawski and L.Ong, E. Beffara, G. Bellin, and recently by Phil Wadler.

Visiting LFCS: 2000-2001

At this time, Samson was at LFCS. Various ideas surrounding linear logic were developing. Two themes during that visit:

1. Bounded LL (BLL) and Implicit Computational Complexity. At LFCS, this involved Martin Hofmann, Patrick Baillot.

   Martin had just finished his Habilitation in logic and implicit complexity theory (ICC). We wrote a paper on *Realizability models for BLL-like languages*, giving a new proof that the functions computable in BLL = ptime functions. Used realizability based on ptime BCK algebras.
   Hofmann, Baillot, dal Lago, et.al. developed ICC into a large and still-active community.

2. Full Completeness, Games Semantics, Geometry of Interaction, Traced Monoidal Categories. *This greatly influenced my research for many years.*

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Logical counterpart to studies in Full Abstraction problem.

Theorem (Laüchli, 1969)

An $\mathcal{L}$ formula A of intuitionistic prop. calculus is provable iff for every interpretation F of the base types in $\mathbb{Z}$-Sets, its interpretation $[\![A]\!]_F$ has an invariant element.

Theorem (Friedman, 1975)

Let $\mathcal{A}$ be a full type hierarchy with base sorts interpreted as infinite sets. Then for pure closed typed lambda terms M, N,

$$\vdash M = N \quad \textit{iff} \quad \mathcal{A} \models M = N$$

Full Completeness: Modelling proofs (= programs)

Given a typed logic $\mathcal{L}$ (or associated free category $\mathcal{F}_{\mathcal{L}}$), we say a model category $\mathcal{M}$ is *fully complete* if , for some interpretation of the base types, the (unique) canonical interpretation

$$\mathcal{F}_{\mathcal{L}} \xrightarrow{[\![-]\!]} \mathcal{M}$$

is full (and, hopefully, faithful). Fullness says $[\![-]\!]$ is *"surjective"*: any $[\![A]\!] \to [\![B]\!] \in \mathcal{M}$ comes from a (hopefully unique) *proof* $\pi : A \vdash B$ in $\mathcal{L}$. (Terminology: Abramsky)

There exist many fully complete models for fragments of LL: **Games** (Abramsky, Hyland, et.al.), **Domains** (Plotkin-Pratt), **Topological V-S models** (Blute-S., Hamano), **GoI** (Haghverdi, Hyland-Schalk).

Main Thing: $\mathcal{M}$ should arise by magic (not related to syntax!). Then full completeness says something surprising & non-trivial!

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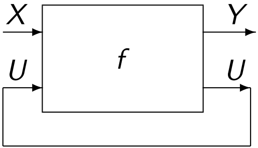
Traced Monoidal Categories, GoI, etc.

LFCS (work of Gordon and Samson) has been central in developing the algebraic theory of feedback. Briefly:

A symmetric monoidal category $(\mathcal{C}, \otimes, I, s)$ with a family of functions Trace

$$Tr^U_{X,Y} : \mathcal{C}(X \otimes U, Y \otimes U) \quad \longrightarrow \quad \mathcal{C}(X, Y)$$



satisfying various naturality and trace equations.

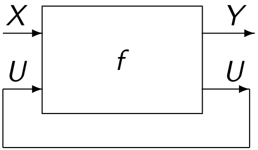
Partially Traced Categories

At MFPS in Montreal in 2003, Gordon Plotkin gave an important talk on defining *partial traces* and associated algebraic machinery.

A detailed development of a notion of partial trace (agreeing with Gordon's) was by E. Haghverdi and P. Scott (2005).

A symmetric monoidal category $(\mathcal{C}, \otimes, I, s)$ with a family of Partial Trace (feedback) functions

$$Tr^{U}_{X,Y} : \mathcal{C}(X \otimes U, Y \otimes U) \rightharpoonup \mathcal{C}(X, Y)$$



with *conditional* naturality & trace equations (wrt Kleene equality).

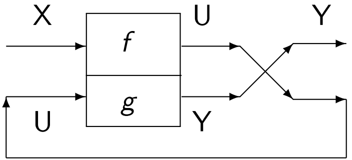
Partially Traced Categories

Theorem (Malherbe, Scott,Selinger (2012), Bagnol (2015))

There is a natural bijective correspondence:
(i) monoidal subcategories $\mathcal{C}$ *of some (totally) traced monoidal category* $\mathcal{D}$ *and (ii) partial traces on* $\mathcal{C}$.

Practical Goal: Above, an ambient (totally traced) $\mathcal{D}$ is built from a partially traced $\mathcal{C}$ by a free/syntactical construction. What about for a *concrete* $\mathcal{C}$? Can we find such a $\mathcal{D}$ in ordinary math?

E.g. for studying feedback in analog computing and related structures in Gordon's MFPS talk, $\mathcal{C}$ = Complete Metric Spaces with a certain partial trace. Is there a concrete $\mathcal{D}$?



$$= \quad X \xrightarrow{f} U \xrightarrow{g} Y$$

$$\mathrm{Tr}^{U}_{X,Y}(s\circ(f \otimes g)) \quad = \quad g\circ f$$

[Says: general composition (i.e. "cut") is definable from more primitive compositions (along symmetries) using a single trace]. Leads to normal form theorem, related to Kleene's NFT in Recursion Theory and the Execution Formula in GoI.

Visiting LFCS 2010–

Have visited LFCS several times on sabbaticals/leaves since 2010.

- In 2010, I was the FLoC 2010 Workshop Chair here (it is hard to say no to Moshe Vardi). Leonid Libkin and I worked like crazy for that FLoC: but it was also a real pleasure to work with the LFCS (and Informatics Staff) in general.

  Leonid gave a fine Lab Lunch talk on the trials and tribulations of our FLoC2010 experience.
- The last two years, I've become interested in Many-Valued Logics (Łukasiewicz , 1920's). In particular, their algebras, so-called MV-algebras, and the work of the logician D. Mundici (Florence) and quantum Effect Algebras (B. Jacobs).

  I began this work in my 2014 sabbatical visit to Informatics. It turned out that Alex Simpson and his student Matteo Mio, had already written on the subject: "Łukasiewicz μ-calculus."

Łukasiewicz Logics and their Algebras (every 30 years)

- Studied by Polish logicians in 1920's, including Lesniewski, Tarski (in parallel with Post (1921) in U.S.)
- 1940's & early 1950's: Rosenbloom, Rosser, McNaughton.
- Mid-1950's: major advances by CC. Chang: MV-algebras, Chang Completeness Thm, lattice ordered abelian groups.
- From mid-1980's: large body of work by D. Mundici, et.al.
  - Connections of MV-Algebras & AF C*-algebras.
  - Connections with works of Elliott, Effros, Handleman: dimension groups and Grothendieck's K_0 functor.
  - States & probability distributions.
- Sheaf Representation of MV-Algebras: Dubuc/Poveda (2010)
- Łukasiewicz μ-calculus, **M. Mio and Alex Simpson** (2013)
- Morita Equivalence of MV-algebras (Caramello, 2014)
- Coordinatization: Lawson & Scott [2014-], F. Wehrung [2015-]

Regrets and the Future?

I regret not making better use of a lot of the extraordinary LFCS expertise:

1. Linguistics
2. Quantum
3. Biology

although I'm looking forward (this Fall) to visiting LFCS yet again, to visit many of the researchers here, and beginning to look at many of these open directions.

Thanks to LFCS for your many kindnesses!