Some Thoughts on LFCS at 30

Samson Abramsky

Department of Computer Science, University of Oxford

Sense And/Or Sensibility?

Sense And/Or Sensibility?

Don's specification:

> I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.

Sense And/Or Sensibility?

Don's specification:

> *I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.*

LFCS has been a major influence:

Sense And/Or Sensibility?

Don's specification:

> *I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.*

LFCS has been a major influence:

- shaping the community in which I work

Sense And/Or Sensibility?

Don's specification:

> *I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.*

LFCS has been a major influence:

- shaping the community in which I work
- directly on my ideas

Sense And/Or Sensibility?

Don's specification:

> *I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.*

LFCS has been a major influence:

- shaping the community in which I work
- directly on my ideas
- as a star to follow, and as a counterpoint to diverge from!

Sense And/Or Sensibility?

Don's specification:

> I'm not expecting a technical talk, rather your thoughts about LFCS
> past, present and/or future, and/or maybe how LFCS and/or work done
> here influenced you and your work.

LFCS has been a major influence:

- shaping the community in which I work
- directly on my ideas
- as a star to follow, and as a counterpoint to diverge from!
  E.g. the work on game semantics, geometry of interaction, interaction
  categories . . .

Sense And/Or Sensibility?

Don's specification:

> *I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.*

LFCS has been a major influence:

- shaping the community in which I work
- directly on my ideas
- as a star to follow, and as a counterpoint to diverge from!
  E.g. the work on game semantics, geometry of interaction, interaction categories . . .
  The support and encouragement I received from Robin, Rod, Gordon, and others at LFCS in following my own path meant a great deal to me.

Sense And/Or Sensibility?

Don's specification:

> *I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.*

LFCS has been a major influence:

- shaping the community in which I work
- directly on my ideas
- as a star to follow, and as a counterpoint to diverge from!
  E.g. the work on game semantics, geometry of interaction, interaction categories . . .
  The support and encouragement I received from Robin, Rod, Gordon, and others at LFCS in following my own path meant a great deal to me.

Many people and ideas in and around LFCS have been important for me!

Sense And/Or Sensibility?

Don's specification:

> *I'm not expecting a technical talk, rather your thoughts about LFCS past, present and/or future, and/or maybe how LFCS and/or work done here influenced you and your work.*

LFCS has been a major influence:

- shaping the community in which I work
- directly on my ideas
- as a star to follow, and as a counterpoint to diverge from! E.g. the work on game semantics, geometry of interaction, interaction categories . . . The support and encouragement I received from Robin, Rod, Gordon, and others at LFCS in following my own path meant a great deal to me.

Many people and ideas in and around LFCS have been important for me!

I shall just pursue one thread.

Before LFCS

Before LFCS

Long before LFCS was established in 1986, there was a pre-existing Theory group in Edinburgh CS.

Before LFCS

Long before LFCS was established in 1986, there was a pre-existing Theory group in Edinburgh CS.

Some key features:

Before LFCS

Long before LFCS was established in 1986, there was a pre-existing Theory group in Edinburgh CS.

Some key features:

- The coming together and cooperation of several remarkable scientific personalities and leaders: Burstall, Milner, Plotkin.

Before LFCS

Long before LFCS was established in 1986, there was a pre-existing Theory group in Edinburgh CS.

Some key features:

- The coming together and cooperation of several remarkable scientific personalities and leaders: Burstall, Milner, Plotkin.
- The development of a characteristic style, vision of the subject, and scientific taste.

Before LFCS

Long before LFCS was established in 1986, there was a pre-existing Theory group in Edinburgh CS.

Some key features:

- The coming together and cooperation of several remarkable scientific personalities and leaders: Burstall, Milner, Plotkin.
- The development of a characteristic style, vision of the subject, and scientific taste.
- An exemplar and inspiration for others.

Before LFCS

Long before LFCS was established in 1986, there was a pre-existing Theory group in Edinburgh CS.

Some key features:

- The coming together and cooperation of several remarkable scientific personalities and leaders: Burstall, Milner, Plotkin.
- The development of a characteristic style, vision of the subject, and scientific taste.
- An exemplar and inspiration for others.
  Hard to grasp now how important that was then!

My first Close Encounter

My first Close Encounter

Working at GEC Computers, 1977, on operating system development. Came across two **orange** booklets:

- George Milne and Robin Milner, *Concurrent Processes and their Syntax*
- Robin Milner, *Flowgraphs and Flow Algebras*.

My first Close Encounter

Working at GEC Computers, 1977, on operating system development. Came across two **orange** booklets:

- George Milne and Robin Milner, *Concurrent Processes and their Syntax*
- Robin Milner, *Flowgraphs and Flow Algebras*.

This was the opening quotation from Flowgraphs and Flow Algebras:

> "You could get an infinite number of wires in this . . . junction box, but we don't usually go that far in practice"
> – Man from London Electricity Board, 1959.

My first Close Encounter

Working at GEC Computers, 1977, on operating system development. Came across two **orange** booklets:

- George Milne and Robin Milner, *Concurrent Processes and their Syntax*
- Robin Milner, *Flowgraphs and Flow Algebras*.

This was the opening quotation from Flowgraphs and Flow Algebras:

> "You could get an infinite number of wires in this . . . junction box, but we don't usually go that far in practice"
> – Man from London Electricity Board, 1959.

It presents an algebra of nets, and proves its completeness under a simple equational theory, using the methods of initial algebra semantics (ADJ).

My first Close Encounter

Working at GEC Computers, 1977, on operating system development. Came across two **orange** booklets:

- George Milne and Robin Milner, *Concurrent Processes and their Syntax*
- Robin Milner, *Flowgraphs and Flow Algebras*.

This was the opening quotation from Flowgraphs and Flow Algebras:

> "You could get an infinite number of wires in this ... junction box, but we don't usually go that far in practice"
> – Man from London Electricity Board, 1959.

It presents an algebra of nets, and proves its completeness under a simple equational theory, using the methods of initial algebra semantics (ADJ).

There are many **pictures** of nets: precursor of many subsequent diagrammatic calculi.

Concurrent Processes and their Syntax

Concurrent Processes and their Syntax

*Concurrency among computations which communicate with each other presents some of the most exciting challenges in the study of computation. Not only is it of great practical relevance, but it also appears to demand a theory of its own, which at present exists only tentatively. One aspect of this theory must be quantitative .... Another (perhaps **the** other) aspect is qualitative; what conceptual framework is needed to understand and discuss the behavior of concurrent computations? Although these two aspects should presumably interact, it appears to be sound methodology to separate them initially.*
This paper deals with the qualitative aspect. We attempt to provide a part of the necessary conceptual framework by attacking the question: What sort of mathematical object is a process?

The Cunning of Reason I

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

- Probability

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

- Probability
- Weighted systems of various kinds

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

- Probability
- Weighted systems of various kinds
- Real-time

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

- Probability
- Weighted systems of various kinds
- Real-time
- Quantum Information

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

- Probability
- Weighted systems of various kinds
- Real-time
- Quantum Information
- Computational Systems Biology

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

- Probability
- Weighted systems of various kinds
- Real-time
- Quantum Information
- Computational Systems Biology
- Computational Game Theory and Economics

The Cunning of Reason I

What is a fruitful guiding principle at a certain stage in the development of a scientific field does not always stay that way!

This separation of concerns, precursor of

- Track A vs. Track B
- Eurotheory vs. US theory

was an important and necessary step in its time.

As was foreseen in the quotation, these two strands, quantitive and qualitative, must eventually interact, and are increasingly doing so:

- Probability
- Weighted systems of various kinds
- Real-time
- Quantum Information
- Computational Systems Biology
- Computational Game Theory and Economics
- and more . . .

The Cunning of Reason II

The Cunning of Reason II

Another important feature, part of the spirit of LFCS:

The Cunning of Reason II

Another important feature, part of the spirit of LFCS:

Explicitly marking Computer Science as a serious scientific discipline in its own right; not just a service subject, an auxiliary, a branch of engineering . . .

The Cunning of Reason II

Another important feature, part of the spirit of LFCS:

Explicitly marking Computer Science as a serious scientific discipline in its own right; not just a service subject, an auxiliary, a branch of engineering ...

Crucial for me, when I picked up those orange booklets:

> "Science is nothing more that doing your damnedest with your mind, no holds barred" Percy W Bridgman

The Cunning of Reason II

Another important feature, part of the spirit of LFCS:

Explicitly marking Computer Science as a serious scientific discipline in its own right; not just a service subject, an auxiliary, a branch of engineering . . .

Crucial for me, when I picked up those orange booklets:

> "Science is nothing more that doing your damnedest with your mind, no holds barred" Percy W Bridgman

Again, the picture looks different now. CS has arrived on the scene; now we must **avoid** remaining in splendid isolation!

The Cunning of Reason II

Another important feature, part of the spirit of LFCS:

Explicitly marking Computer Science as a serious scientific discipline in its own right; not just a service subject, an auxiliary, a branch of engineering ...

Crucial for me, when I picked up those orange booklets:

> "Science is nothing more that doing your damnedest with your mind, no holds barred" Percy W Bridgman

Again, the picture looks different now. CS has arrived on the scene; now we must **avoid** remaining in splendid isolation!

More positively: exciting things are happening on the boundaries with other disciplines. The methods and modes of thought of CS **can** – and **should** – be applied much more widely.

The Cunning of Reason II

Another important feature, part of the spirit of LFCS:

Explicitly marking Computer Science as a serious scientific discipline in its own right; not just a service subject, an auxiliary, a branch of engineering ...

Crucial for me, when I picked up those orange booklets:

> "Science is nothing more that doing your damnedest with your mind, no holds barred" Percy W Bridgman

Again, the picture looks different now. CS has arrived on the scene; now we must **avoid** remaining in splendid isolation!

More positively: exciting things are happening on the boundaries with other disciplines. The methods and modes of thought of CS **can** – and **should** – be applied much more widely.

Cautionary tales: LiCS and other battlegrounds ...

The Cunning of Reason III

The Cunning of Reason III
What do we need to know?

The Cunning of Reason III

What do we need to know?

Those papers by Robin defined an initial syllabus for me.

The Cunning of Reason III
What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

The Cunning of Reason III

What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

How it looked back then: logic, order theory, universal algebra, some (general/point-set) topology, increasingly category theory . . .

The Cunning of Reason III
What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

How it looked back then: logic, order theory, universal algebra, some (general/point-set) topology, increasingly category theory . . .

A clear remove from "mainstream mathematics"

The Cunning of Reason III

What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

How it looked back then: logic, order theory, universal algebra, some (general/point-set) topology, increasingly category theory . . .

A clear remove from "mainstream mathematics"

Another aspect of the separation of qualitative from quantitative.

The Cunning of Reason III

What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

How it looked back then: logic, order theory, universal algebra, some (general/point-set) topology, increasingly category theory . . .

A clear remove from "mainstream mathematics"

Another aspect of the separation of qualitative from quantitative.

It looks very different now!

The Cunning of Reason III

What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

How it looked back then: logic, order theory, universal algebra, some (general/point-set) topology, increasingly category theory . . .

A clear remove from "mainstream mathematics"

Another aspect of the separation of qualitative from quantitative.

It looks very different now!

My own (modest, call-by-need) experience from quantum information and foundations.

The Cunning of Reason III

What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

How it looked back then: logic, order theory, universal algebra, some
(general/point-set) topology, increasingly category theory . . .

A clear remove from "mainstream mathematics"

Another aspect of the separation of qualitative from quantitative.

It looks very different now!

My own (modest, call-by-need) experience from quantum information and
foundations.

Mathematics which has been **unavoidable**:
probability, linear algebra, functional analysis, linear programming, convex
geometry, projective geometry, logic, algorithms and complexity, sheaves and
cohomology, . . .

The Cunning of Reason III

What do we need to know?

Those papers by Robin defined an initial syllabus for me.
Later, I learned domain theory by reading Gordon's papers (!!)

How it looked back then: logic, order theory, universal algebra, some (general/point-set) topology, increasingly category theory . . .

A clear remove from "mainstream mathematics"

Another aspect of the separation of qualitative from quantitative.

It looks very different now!

My own (modest, call-by-need) experience from quantum information and foundations.

Mathematics which has been **unavoidable**:
probability, linear algebra, functional analysis, linear programming, convex geometry, projective geometry, logic, algorithms and complexity, sheaves and cohomology, . . .

Potentially, there is much more. Look at the students!

Show, don't tell

Show, don't tell

I will illustrate these themes:

Show, don't tell

I will illustrate these themes:

- Reconnecting the qualitative and structural with the quantitative

Show, don't tell

I will illustrate these themes:

- Reconnecting the qualitative and structural with the quantitative
- CS interconnecting with other fields of science, being influenced by – and daring to (try to) influence!

Show, don't tell

I will illustrate these themes:

- Reconnecting the qualitative and structural with the quantitative
- CS interconnecting with other fields of science, being influenced by – and daring to (try to) influence!
- Broadening the mathematical tools, connecting with the "mainstream"

Illustrate literally, with some pictures to give an impression of some current work on **contextuality**

Show, don't tell

I will illustrate these themes:

- Reconnecting the qualitative and structural with the quantitative
- CS interconnecting with other fields of science, being influenced by – and daring to (try to) influence!
- Broadening the mathematical tools, connecting with the "mainstream"

Illustrate literally, with some pictures to give an impression of some current work on **contextuality**

Rui Soares Barbosa, Kohei Kishida, Ray Lal and Shane Mansfield

Show, don't tell

I will illustrate these themes:

- Reconnecting the qualitative and structural with the quantitative
- CS interconnecting with other fields of science, being influenced by – and daring to (try to) influence!
- Broadening the mathematical tools, connecting with the "mainstream"

Illustrate literally, with some pictures to give an impression of some current work on **contextuality**

Rui Soares Barbosa, Kohei Kishida, Ray Lal and Shane Mansfield

The slogan: contextuality arises where we have a family of data which is **locally consistent**, but **globally inconsistent**.

Contextuality Analogy: Local Consistency
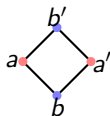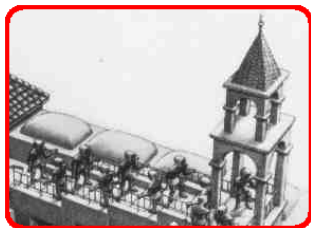
Contextuality Analogy: Local Consistency
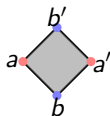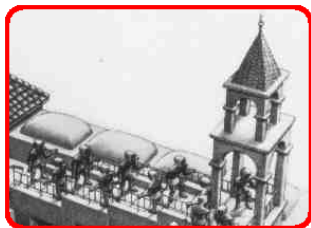
Contextuality Analogy: Global Inconsistency

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
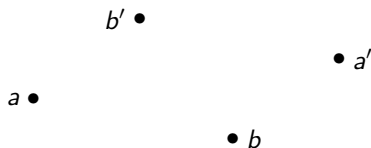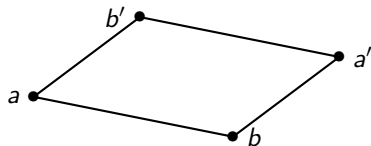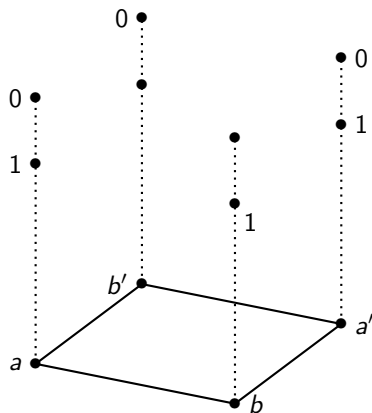- E.g. the Hardy model:

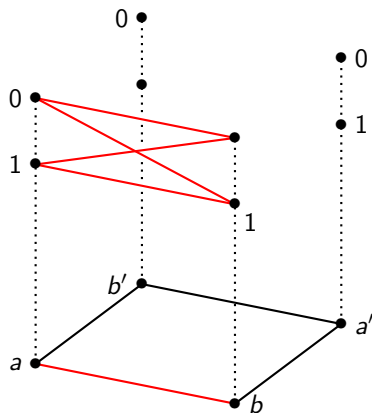|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

$b' \bullet$

$\bullet\, a'$
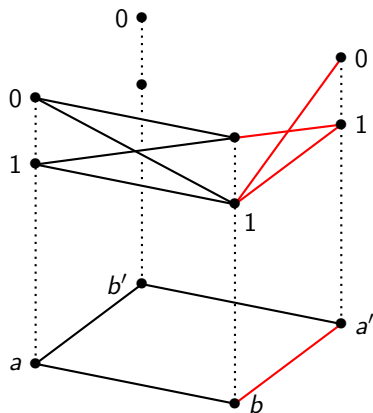
$a \, \bullet$

$\bullet\, b$

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

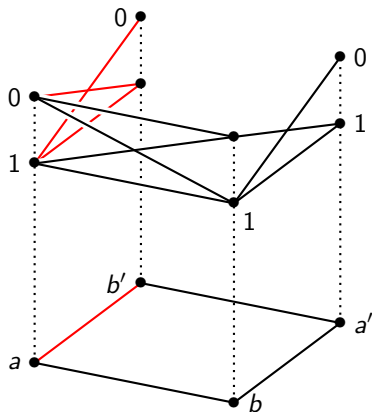|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

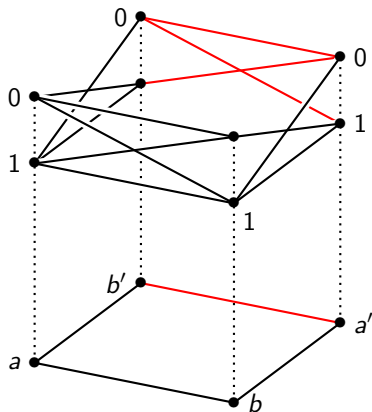|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

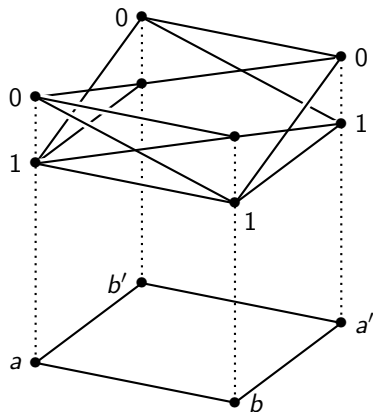|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

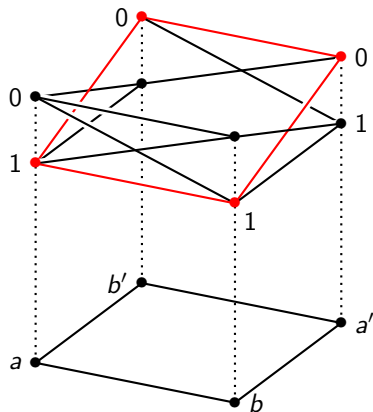|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

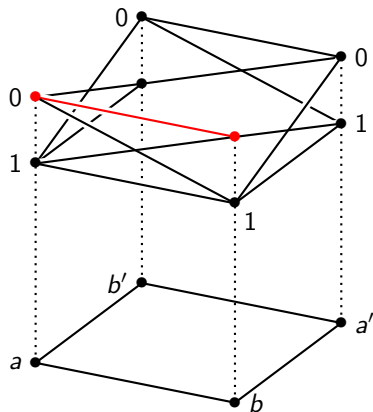|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

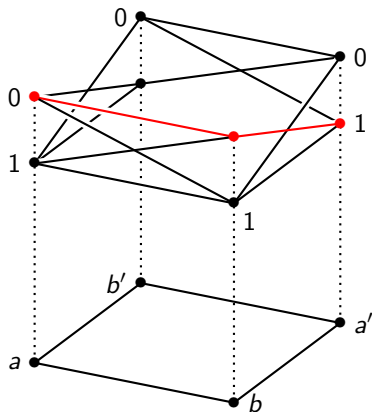|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

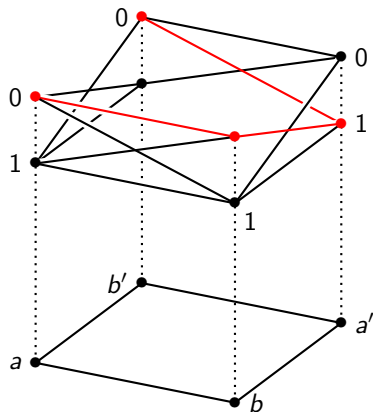|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

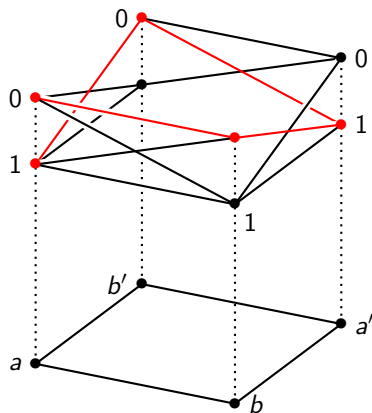| || 00 | 01 | 10 | 11 |
--------		----	----	----	----
ab		✓	✓	✓	✓
ab'		×	✓	✓	✓
$a'b$		×	✓	✓	✓
$a'b'$		✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

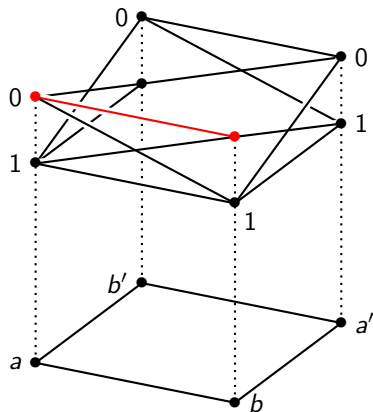|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

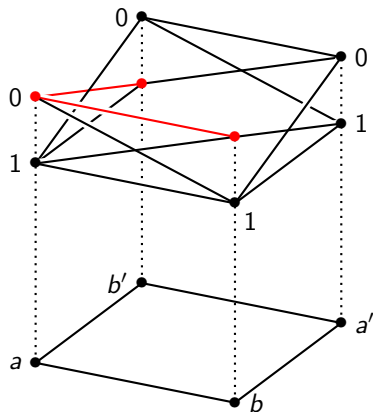|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

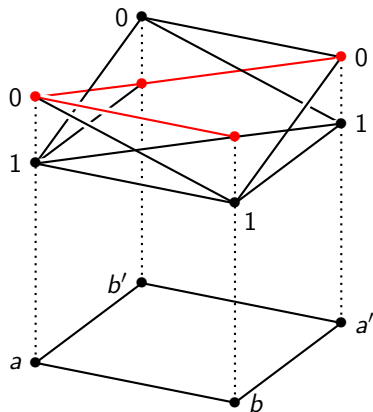|	00	01	10	11
ab	✓	✓	✓	✓
ab'	✕	✓	✓	✓
$a'b$	✕	✓	✓	✓
$a'b'$	✓	✓	✓	✕

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

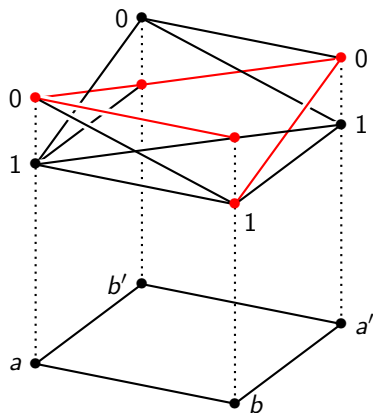|	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Logical Contextuality

- Ignore precise probabilities
- Events are possible or not
- E.g. the Hardy model:

	00	01	10	11
ab	✓	✓	✓	✓
ab'	×	✓	✓	✓
$a'b$	×	✓	✓	✓
$a'b'$	✓	✓	✓	×

Bundle Pictures

Strong Contextuality

- E.g. the PR box:

	00	01	10	11
ab	✓	×	×	✓
ab'	✓	×	×	✓
$a'b$	✓	×	×	✓
$a'b'$	×	✓	✓	×
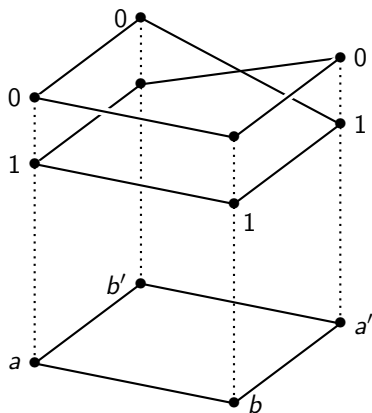
Contextuality, Logic and Paradoxes

Contextuality, Logic and Paradoxes

Liar cycles. A Liar cycle of length N is a sequence of statements

$$S_1 : \ S_2 \text{ is true,}$$
$$S_2 : \ S_3 \text{ is true,}$$
$$\vdots$$
$$S_{N-1} : \ S_N \text{ is true,}$$
$$S_N : \ S_1 \text{ is false.}$$

For $N = 1$, this is the classic Liar sentence

$$S : \ S \text{ is false.}$$

Contextuality, Logic and Paradoxes

Liar cycles. A Liar cycle of length N is a sequence of statements

$$S_1 : \; S_2 \text{ is true,}$$
$$S_2 : \; S_3 \text{ is true,}$$
$$\vdots$$
$$S_{N-1} : \; S_N \text{ is true,}$$
$$S_N : \; S_1 \text{ is false.}$$

For $N = 1$, this is the classic Liar sentence

$$S : \; S \text{ is false.}$$

Following Cook, Walicki et al. we can model the situation by boolean equations:

$$x_1 = x_2, \; \ldots, \; x_{n-1} = x_n, \; x_n = \neg x_1$$

Contextuality, Logic and Paradoxes

Liar cycles. A Liar cycle of length N is a sequence of statements

$$S_1 : \ S_2 \text{ is true},$$
$$S_2 : \ S_3 \text{ is true},$$
$$\vdots$$
$$S_{N-1} : \ S_N \text{ is true},$$
$$S_N : \ S_1 \text{ is false}.$$

For $N = 1$, this is the classic Liar sentence

$$S : \ S \text{ is false}.$$

Following Cook, Walicki et al. we can model the situation by boolean equations:

$$x_1 = x_2, \ \ldots, \ x_{n-1} = x_n, \ x_n = \neg x_1$$

The "paradoxical" nature of the original statements is now captured by the inconsistency of these equations.

Contextuality in the Liar; Liar cycles in the PR Box

Contextuality in the Liar; Liar cycles in the PR Box

We can regard each of these equations as fibered over the set of variables which occur in it:

$$\{x_1, x_2\} : \quad x_1 = x_2$$
$$\{x_2, x_3\} : \quad x_2 = x_3$$
$$\vdots$$
$$\{x_{n-1}, x_n\} : \quad x_{n-1} = x_n$$
$$\{x_n, x_1\} : \quad x_n = \neg x_1$$

Contextuality in the Liar; Liar cycles in the PR Box

We can regard each of these equations as fibered over the set of variables which occur in it:

$$\{x_1, x_2\} : \quad x_1 = x_2$$
$$\{x_2, x_3\} : \quad x_2 = x_3$$
$$\vdots$$
$$\{x_{n-1}, x_n\} : \quad x_{n-1} = x_n$$
$$\{x_n, x_1\} : \quad x_n = \neg x_1$$

Any subset of up to $n-1$ of these equations is consistent; while the whole set is inconsistent.

Contextuality in the Liar; Liar cycles in the PR Box

We can regard each of these equations as fibered over the set of variables which occur in it:

$$\{x_1, x_2\}: \quad x_1 = x_2$$
$$\{x_2, x_3\}: \quad x_2 = x_3$$
$$\vdots$$
$$\{x_{n-1}, x_n\}: \quad x_{n-1} = x_n$$
$$\{x_n, x_1\}: \quad x_n = \neg x_1$$

Any subset of up to $n - 1$ of these equations is consistent; while the whole set is inconsistent.

Up to rearrangement, **the Liar cycle of length 4 corresponds exactly to the PR box**.

Contextuality in the Liar; Liar cycles in the PR Box

We can regard each of these equations as fibered over the set of variables which occur in it:

$$
\begin{aligned}
\{x_1, x_2\} : \quad x_1 &= x_2 \\
\{x_2, x_3\} : \quad x_2 &= x_3 \\
&\vdots \\
\{x_{n-1}, x_n\} : \quad x_{n-1} &= x_n \\
\{x_n, x_1\} : \quad x_n &= \neg x_1
\end{aligned}
$$

Any subset of up to $n - 1$ of these equations is consistent; while the whole set is inconsistent.

Up to rearrangement, **the Liar cycle of length 4 corresponds exactly to the PR box**.

The usual reasoning to derive a contradiction from the Liar cycle corresponds precisely to the attempt to find a univocal path in the bundle diagram.

Paths to contradiction
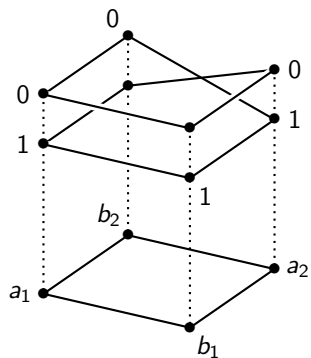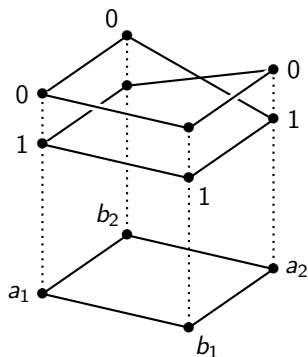
Paths to contradiction
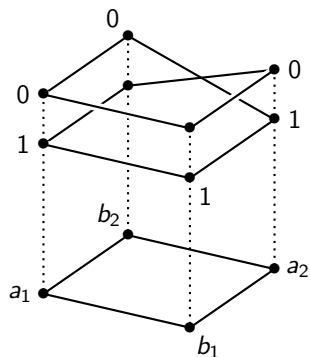
Suppose that we try to set a_2 to 1. Following the path on the right leads to the following local propagation of values:

$$a_2 = 1 \rightsquigarrow b_1 = 1 \rightsquigarrow a_1 = 1 \rightsquigarrow b_2 = 1 \rightsquigarrow a_2 = 0$$
$$a_2 = 0 \rightsquigarrow b_1 = 0 \rightsquigarrow a_1 = 0 \rightsquigarrow b_2 = 0 \rightsquigarrow a_2 = 1$$

Paths to contradiction

Suppose that we try to set a_2 to 1. Following the path on the right leads to the following local propagation of values:

$$a_2 = 1 \rightsquigarrow b_1 = 1 \rightsquigarrow a_1 = 1 \rightsquigarrow b_2 = 1 \rightsquigarrow a_2 = 0$$
$$a_2 = 0 \rightsquigarrow b_1 = 0 \rightsquigarrow a_1 = 0 \rightsquigarrow b_2 = 0 \rightsquigarrow a_2 = 1$$

We have discussed a specific case here, but the analysis can be generalised to a large class of examples.

Topological Characterisation

- Čech cohomology
  (on a presheaf derived from the model)
- Topological invariant $\gamma(s)$ for every possible event s
- $\gamma(s) \neq \emptyset$ implies contextuality
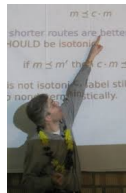- Cohomological witnesses for PR box, GHZ, Peres-Mermin, Specker's triangle, Kochen-Specker, All AvN models, . . .
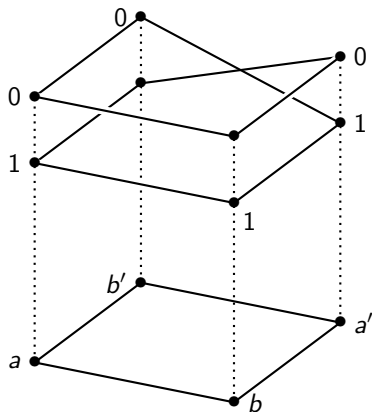
Edinburgh Memories

Edinburgh Memories

The Edinburgh interaction group

Edinburgh Memories

The Edinburgh interaction group

Kohei Honda, Nobuko Yoshida, Paul-Andé Melliès, Juliusz Chroboczek, Jim Laird

A Parting Shot

Some good advice:

> "Avoid the temptation to work so hard that there is no time left for serious thinking". Francis Crick

A Parting Shot

Some good advice:

> "Avoid the temptation to work so hard that there is no time left for serious thinking". Francis Crick

Happy Anniversary LFCS!